

# Four-year COBE normalization of inflationary cosmologies

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We supply fitting formulae enabling the normalization of slow-roll inflation models to the four-year COBE data. We fully include the effect of the gravitational wave modes, including the predicted relation of the amplitude of these modes to that of the density perturbations. We provide the normalization of the matter power spectrum, which can be directly used for large-scale structure studies. The normalization for tilted spectra is a special case. We also provide fitting functions for the inflationary energy scale of COBE-normalized models and discuss the validity of approximating the spectra by power-laws. In an Appendix, we extend our analysis to include models with a cosmological constant, both with and without gravitational waves.

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## I. INTRODUCTION

The four-year data set from the Cosmic Background Explorer (COBE) satellite [1] is the last word we shall hear for some time concerning irregularities on the largest observable scales. One of the most important uses of the COBE data is in providing an accurate normalization of the power spectrum of density perturbations for a given set of theoretical assumptions; once normalized, one can then compare the theory to a wide range of observations of large-scale structure in the Universe.

The aim of this short paper is to apply the techniques of Bunn and White [2] to normalize slow-roll inflation models. As well as a spectrum of density perturbations, inflationary models predict a spectrum of gravitational waves, which can influence the large-angle microwave anisotropies seen by COBE. Further, in inflationary models one predicts a generic link between the density perturbations and gravitational waves, which must be taken into account in obtaining an accurate normalization. In this paper, we provide a self-contained account of how to predict these spectra from an inflationary model and quote fits to COBE for the normalization of the matter power spectrum, which can be directly used for large-scale structure studies. We also give fitting functions for the inflationary energy scale.

## II. SPECTRA FROM INFLATION

We are of course unable to provide information for an arbitrary inflationary model. We shall not consider models with more than one dynamical field, for which the

calculations are extremely involved [3] and for which initial conditions may be important, and we are also unable to consider open inflationary models [4] because in that case no-one has yet managed to compute the gravitational wave spectrum. We therefore restrict ourselves to the usual single-field inflation models, driven by a rolling scalar field  $\phi$ , known generically as chaotic inflation [5]. This situation is a very general one, because most two-field models, for example hybrid inflation [6], feature only a single dynamical scalar field, and models with extended gravity sectors can usually be brought into the Einstein form via conformal transformation [7].

Exact results for the spectra are not known for arbitrary potentials  $V(\phi)$ . They can be calculated analytically via the slow-roll approximation. The accuracy required depends on the way in which the results shall be used, and for normalizing to COBE it is valid to use the well known lowest-order results (see, e.g., Ref. [8]), which give the density perturbation (i.e. scalar) spectrum  $A_S(k)$  and gravitational wave (i.e. tensor) spectrum  $A_T(k)$  as

$$A_S^2(k) = \frac{512\pi}{75} \left. \frac{V^3}{m_{\text{Pl}}^6 V'^2} \right|_{k=aH}, \quad (1)$$

$$A_T^2(k) = \frac{32}{75} \left. \frac{V}{m_{\text{Pl}}^4} \right|_{k=aH}, \quad (2)$$

where prime indicates derivative with respect to  $\phi$ , and the right-hand side is to be evaluated at the  $\phi$  value when the scale  $k$  equals the Hubble scale during inflation. The precise definition of the spectra will be clarified later.

More accurate expressions than these do exist in the literature [9]. However, these are *not* necessary for discussion of the COBE normalization, though they will be needed for discussion of the inflationary energy scale, as

discussed later.

The slow-roll approximation is characterized by the smallness (relative to unity) of two parameters\* [11,8]

$$\epsilon \equiv \frac{m_{\text{Pl}}^2}{16\pi} \left( \frac{V'}{V} \right)^2, \quad \eta \equiv \frac{m_{\text{Pl}}^2}{8\pi} \frac{V''}{V}. \quad (3)$$

Using Eq. (8) below, the spectral indices of the two spectra can be written in terms of these parameters

$$n - 1 \equiv \frac{d \ln A_{\text{S}}^2(k)}{d \ln k} = -6\epsilon + 2\eta, \quad (4)$$

$$n_{\text{T}} \equiv \frac{d \ln A_{\text{T}}^2(k)}{d \ln k} = -2\epsilon, \quad (5)$$

as can the ratio of the two spectra

$$\frac{A_{\text{T}}^2}{A_{\text{S}}^2} = \epsilon = -\frac{n_{\text{T}}}{2}. \quad (6)$$

Note this ratio is not independent of the tensor spectral index; the gravitational wave and density perturbation spectra are related due to their common origin in a single potential  $V(\phi)$  [11].<sup>†</sup>

All the above expressions apply at any scale  $k$ , with the spectral indices able to vary with scale. Since COBE covers a fairly restricted range of scales, the spectra produced can be approximated by power-laws. Then we need to specify the amplitudes and spectral indices only at a single scale. It is best to choose this scale near the center of the COBE data, so we evaluate them at the scale  $k_* = 7a_0H_0$ . Since, loosely speaking, the  $\ell$ -th microwave multipole samples scales around  $k = \ell a_0H_0/2$ , this corresponds to the fourteenth multipole. Throughout, subscript ‘\*’ will indicate evaluation at this scale, and subscript ‘0’ indicates present value, here of the scale factor  $a$  and Hubble parameter  $H$ .

For a specification of the COBE normalization to have a precise meaning, we need to take care in relating scales during inflation to present scales. The number of  $e$ -foldings  $N$  before the end of inflation at which  $k = aH$  is given by (see, e.g., Ref. [8])

$$N(k) = 62 - \ln \frac{k}{a_0H_0} - \ln \frac{10^{16}\text{GeV}}{V_k^{1/4}} + \ln \frac{V_k^{1/4}}{V_{\text{end}}^{1/4}} - \frac{1}{3} \ln \frac{V_{\text{end}}^{1/4}}{\rho_{\text{reh}}}. \quad (7)$$

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\*Beware that two slightly different versions of these exist in the literature, depending on whether the fundamental quantity is taken to be the potential or the Hubble parameter [10]. We use the former.

<sup>†</sup>Only the spectra are related. The phases within a given realization are uncorrelated.

Here  $V_k$  is the potential when  $k = aH$ ,  $V_{\text{end}}$  is the potential at the end of inflation and  $\rho_{\text{reh}}$  is the energy density immediately after reheating has completed, resuming standard big bang evolution.

The appropriate point on the inflationary potential to evaluate the spectra is given by  $N_*$ . It depends on the energy scale of inflation, which itself depends on the normalization, fortunately only weakly. Once the COBE normalization is found,  $V_*$  and  $V_{\text{end}}$  are determined in the context of a specific model, and the normalization can be iteratively improved if desired to take these values into account. However, the reheat energy is much more uncertain, and so consequently  $N_*$  is not normally specified very accurately. Often,  $N_*$  is taken to be 60 or 50.

To locate the  $\phi$ -value when  $k_* = aH$  during inflation, one simply carries out the integral

$$N(\phi) \simeq \frac{8\pi}{m_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi, \quad (8)$$

where  $\phi_{\text{end}}$  could be calculated numerically but is normally given to adequate accuracy by the breakdown of the slow-roll conditions, taken as  $\epsilon_{\text{end}} = 1$ .<sup>‡</sup> Having located  $\phi_*$ , one immediately gets the slow-roll parameters and hence the spectral indices at that scale. The tensor spectral index  $n_{\text{T}}$ , which is the hardest thing to directly observe, can be eliminated through its relation to the ratio  $A_{\text{T}}^2/A_{\text{S}}^2$ . To indicate the amount of tensors, we define a quantity  $r$  by [11,8]

$$r = 12.4 \frac{A_{\text{T}}^2(k_*)}{A_{\text{S}}^2(k_*)}, \quad (9)$$

which measures, in the matter-dominated and Sachs-Wolfe approximations, the relative importance of gravitational waves and density perturbations in contributing to the relevant microwave multipole, in this case the fourteenth. Henceforth,  $n$  and  $n_{\text{T}}$  will also be assumed to be evaluated at  $k_*$ .

### III. FITTING TO THE FOUR-YEAR COBE DATA

#### A. Normalization of the power spectrum

Large-scale structure studies require the normalization of the present-day power spectrum. We precisely define our notation for the initial spectrum  $A_{\text{S}}(k)$  here. In a critical-density universe, it is related to the rms fluctuation per logarithmic  $k$ -interval  $\Delta^2(k)$ , and to the usual power spectrum  $P(k)$ , both at the present epoch, by

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<sup>‡</sup>In models where inflation doesn't end by steepening of the potential, such as hybrid inflation [6],  $\phi_{\text{end}}$  is given by a different condition such as an instability condition.

$$\Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2} \equiv \left( \frac{k}{a_0 H_0} \right)^4 A_S^2(k) T^2(k), \quad (10)$$

where  $T(k)$  is the usual transfer function, normalized to unity on large scales. The variance of the density field, smoothed on scale  $R$ , is given by

$$\sigma^2(R) = \int \frac{dk}{k} \Delta^2(k) W^2(kR), \quad (11)$$

where the smoothing function  $W(kR)$  tends to unity at small  $k$ . The observables related to  $A_T$  are discussed in Refs. [12].

The fitting to COBE is described in Ref. [2]. Rather than use Eq. (6) to set the relative normalization of scalars and tensors, we use the more accurate expression [13,12]

$$\frac{A_T^2(k_*)}{A_S^2(k_*)} = -\frac{n_T}{2} \left( 1 - \frac{n_T}{2} + (n-1) \right). \quad (12)$$

This relation comes from a full higher-order calculation, so higher-order expressions can also be used if desired to compute  $n$  and  $n_T$  from a given inflation model, along the lines of Refs. [9,13]. Using this,  $r$  is related to the spectral indices by

$$r = -6.2n_T \left( 1 - \frac{n_T}{2} + (n-1) \right). \quad (13)$$

We specify the normalization of the density perturbations, following Ref. [2], at the present Hubble scale  $k = a_0 H_0$ ,<sup>§</sup> and define

$$\delta_H \equiv A_S(a_0 H_0). \quad (14)$$

By focusing on the normalization at such a large scale, we obtain a result which is independent, to excellent accuracy, of the choice of cosmological parameters, such as the present Hubble constant and the nature of the dark matter.\*\* The one exception is a nonzero cosmological constant, which does affect the large-angle anisotropies; we generalize our results to that case in the Appendix.

Our main result is a fitting function for the COBE normalization, which is accurately represented by

$$\delta_H(n, r) = 1.91 \times 10^{-5} \frac{\exp[1.01(1-n)]}{\sqrt{1+0.75r}}. \quad (15)$$

The  $1\sigma$  observational error is 7%. Within the region  $0.7 \leq n \leq 1.3$  and  $-0.3 \leq n_T \leq 0$  (the latter corresponding to  $0 \leq r \lesssim 2$ ), the fit is good to within 1.5%

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<sup>§</sup>It is irrelevant that this differs from the scale at which we evaluated the spectra to do the normalization. The two scales are simply related,  $A_S^2(a_0 H_0) = 7^{1-n} A_S^2(7a_0 H_0)$ .

\*\*For the record, we take pure cold dark matter with  $h = 0.75$  and  $\Omega_{\text{baryon}} h^2 = 0.0125$ .

everywhere. The change from varying other cosmological parameters (except the density parameter) is within 4% for reasonable variations. In addition, there is a systematic uncertainty of  $\sim 3\%$  associated with the process by which the COBE maps are made. Combining all of these uncertainties in quadrature, we believe that a realistic estimate of the uncertainty in  $\delta_H$  is 9% at  $1\sigma$ .

The terms in Eq. (15) have a simple interpretation. The numerical prefactor is the result for a scale-invariant density perturbation spectrum. The  $n$  term represents the pivot point of the COBE data; it guarantees that COBE normalized spectra at fixed  $r$  cross at  $k_{\text{pivot}} = e^{2.02} a_0 H_0$ . This number actually corresponds to about the fifteenth multipole (and in fact a purely scalar fit even prefers the sixteenth), but the tensors give greater weight to the lower multipoles, making  $\ell = 14$  a better overall choice for the pivot. The pivot point is at higher  $\ell$  in the four-year data than in the two-year data, since the low multipoles were already cosmic-variance limited and the higher  $\ell$  have improved signal-to-noise ratio.

The final term in Eq. (15) is the reduction in amplitude due to the tensors. It is interesting that the tensor term has a coefficient of only 0.75, since the definition of  $r$  was intended to make that coefficient close to unity. However, the factor 12.4 in Eq. (9) was computed [11] using the fully matter-dominated Sachs-Wolfe approximation for both the tensor and scalar spectra. The dominant correction to this is the effect on the tensor spectrum from the universe not being perfectly matter dominated at last scattering [12] (about twenty percent), with the start of the rise to the acoustic peak from the density perturbations contributing another five percent. These corrections have been noted in papers concerned with ‘cosmic confusion’ [14]. That the coefficient is only 0.75 means that papers using the original analytic argument have somewhat over-estimated the amount by which tensors reduce the power spectrum normalization.

## B. The inflationary energy scale

For a given inflationary model, the COBE normalization fixes the energy scale of inflation at that time,  $V_*$ . The fitting function can therefore be inverted to supply the inflationary energy scale [15,12]. One would like to use Eq. (1) in order to do this, but in fact this equation is not always very accurate at determining the amplitude, since the slow-roll approximation is not necessarily as accurate as the observations and fits we have discussed. We therefore use the next-order version of this equation, as derived by Stewart and Lyth [9], which is<sup>††</sup>

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<sup>††</sup>To derive this from their results, some further relations are necessary which can be found in Ref. [10].

$$A_S^2(k) = [1 + 4.0\epsilon - 2.1\eta] \frac{512\pi}{75} \frac{V^3}{m_{\text{Pl}}^6 V'^2} \Big|_{k=aH}. \quad (16)$$

It is vital to note that we did not have to use this for the normalization, because the amplitude correction in the prefactor is typically almost constant across the COBE scales (that is,  $\epsilon$  and  $\eta$  hardly vary). It therefore cancels out when one does the COBE normalization for large-scale structure.

We specify the energy at  $k_*$ , the place where the slow-roll parameters were evaluated. Since  $A^2(k_*) = 7^{n-1} \delta_H^2$ , substituting in the fitting function and carrying out a small parameter expansion gives

$$\begin{aligned} V_*^{1/4} &= (5.4 \times 10^{-3} m_{\text{Pl}}) \epsilon_*^{1/4} (1 - 3.2\epsilon_* + 0.5\eta_*) , \\ &= (6.6 \times 10^{16} \text{ GeV}) \epsilon_*^{1/4} (1 - 3.2\epsilon_* + 0.5\eta_*) . \end{aligned} \quad (17)$$

Even if one does not have a specific inflation model in mind, it is possible to use this to obtain an upper bound on the energy density at the end of inflation by imposing some assumptions about how inflation will end [16].

### C. On the validity of the power-law approximation

It is possible to test the validity of approximating the spectra by power-laws. We'll concentrate on the density perturbations. To a first approximation, variation in the spectral indices is driven by the difference between  $n - 1$  and  $n_T$ , but one should consider the full slow-roll formula for that variation at  $k_*$  which is [17]

$$\frac{dn}{d \ln k} \Big|_* = -24\epsilon_*^2 + 16\epsilon_*\eta_* - \frac{m_{\text{Pl}}^4}{32\pi} \frac{V_*' V_*'''}{V_*^2}. \quad (18)$$

The COBE data extend only for slightly more than one log interval in either direction about the central point, so unless this number is greater than, say, a few hundredths, the power-law approximation for COBE will be excellent. Kosowsky and Turner [17] evaluated this for a range of inflation models without finding a value anywhere near this large.

It is possible that the spectra may be well approximated by power-laws on the COBE scales, but not across the much wider range corresponding to future microwave anisotropy observations and to large-scale structure [18,17]. In that case, one simply uses our results with the appropriate approximate power-law at the COBE scales.

### IV. A WORKED EXAMPLE: THE QUADRATIC POTENTIAL

The simplest inflationary model is chaotic inflation [5] with a quadratic potential  $V(\phi) = m^2 \phi^2/2$ . For this potential, the slow-roll parameters are

$$\epsilon = \eta = \frac{1}{4\pi} \frac{m_{\text{Pl}}^2}{\phi^2}, \quad (19)$$

and hence  $\phi_{\text{end}} \simeq m_{\text{Pl}}/\sqrt{4\pi}$ . For definiteness, we take  $N_* = 60$ , and from Eq. (8) we find  $\phi_* = 3.10 m_{\text{Pl}}$ . So for this model one predicts

$$n = 0.967 \quad ; \quad n_T = -0.017 \quad ; \quad r = 0.10 . \quad (20)$$

The normalization of the matter power spectrum, from Eq. (15), is therefore  $\delta_H = 1.91 \times 10^{-5}$ . Although this model is very close to scale-invariant limit, the difference is still non-negligible. For example, if one computes the variance at  $8h^{-1}$  Mpc, denoted  $\sigma_8$ , it is reduced, relative to scale-invariance with no gravitational waves, by 10%, where 6% is due to the tilt and 4% due to the gravitational waves. Many other inflation models give much larger corrections than this.

The inflationary energy scale corresponding to this, from Eq. (17), is

$$V_*^{1/4} = 1.6 \times 10^{-3} m_{\text{Pl}} = 2.0 \times 10^{16} \text{ GeV} , \quad (21)$$

corresponding to  $m = 1.2 \times 10^{-6} m_{\text{Pl}}$ .

Finally, the scale-dependence of the spectral index is  $dn/d \ln k|_* = -5 \times 10^{-4}$ , which is completely negligible for COBE.

## V. SUMMARY

One of the most important uses of the COBE data is to normalize the matter power spectrum used in large-scale structure studies. We have shown that it is possible to condense this information into a single fitting function, covering not just the case of tilted perturbation spectra but also including the spectrum of gravitational waves that inflation predicts. A further extension to models with a cosmological constant is given in the Appendix. Within the slow-roll paradigm, the amount of gravitational waves, parameterized by  $r$ , is completely independent of the density perturbation slope  $n$ , though the scale-dependence of the gravitational wave spectrum is then predicted [11,8]. The fitting functions can also be used for the case without gravitational waves, simply by setting  $r$  equal to zero.

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## APPENDIX A: GENERALIZATION TO MODELS WITH A COSMOLOGICAL CONSTANT

The dynamics of inflation are insensitive to whether or not there is a present cosmological constant  $\Lambda$ , provided the spatial geometry is kept flat. We now provide the generalization of our expressions to the case where  $\Lambda \neq 0$ .

We need a generalization of Eq. (10) to account for the change in the growth of perturbations in a low-density universe, and the relation between the curvature perturbation and the matter power spectrum. If  $\delta_H$  continues to indicate the present power spectrum, and  $A_S$  the initial perturbation spectrum as before, then

$$\delta_H \equiv \frac{g(\Omega_0)}{\Omega_0} A_S(a_0 H_0), \quad (\text{A1})$$

and the right hand side of Eq. (10) is multiplied by  $g^2(\Omega_0)/\Omega_0^2$ . Here  $g(\Omega)$  is the growth suppression factor, which is accurately fit by [19]

$$g(\Omega) = \frac{5}{2} \Omega \left[ \frac{1}{70} + \frac{209\Omega}{140} - \frac{\Omega^2}{140} + \Omega^{4/7} \right]^{-1}. \quad (\text{A2})$$

A generalization of the factor 0.75 multiplying the gravitational wave term in the fitting function for  $\delta_H$  is also needed. By directly evaluating the radiation power spectra for scale-invariant initial spectra with  $A_S = A_T = 1$ , we find the ratio of contributions to  $C_{14}$  is fit to within a percent by

$$\frac{C_{14}^T}{C_{14}^S} \equiv f(\Omega_0) \simeq 0.75 - 0.13 \Omega_\Lambda^2, \quad (\text{A3})$$

with  $\Omega_\Lambda = 1 - \Omega_0$ .

First considering the case with no tensors, a good fit is obtained by using an  $\Omega_0$  dependence plus a cross-term between  $\tilde{n} = n - 1$  and  $\Omega_\Lambda$ . The formula

$$\delta_H(n, \Omega_0) = 1.91 \times 10^{-5} \exp[-1.01 \tilde{n}] \Omega_0^{-0.80-0.05 \ln \Omega_0} \times [1 + 0.18 \tilde{n} \Omega_\Lambda], \quad (\text{A4})$$

holds within 2.5% for  $n$  as before and  $0.2 \leq \Omega_0 \leq 1$ .

We can extend this to include tensors using Eq. (A3) and the addition of an extra cross-term which vanishes in the critical-density case and the tensorless case. This gives

$$\delta_H = 1.91 \times 10^{-5} \frac{\exp[-1.01 \tilde{n}]}{\sqrt{1 + f(\Omega_0) r}} \Omega_0^{-0.80-0.05 \ln \Omega_0} \times [1 + 0.18 \tilde{n} \Omega_\Lambda - 0.03 r \Omega_\Lambda]. \quad (\text{A5})$$

For the parameter ranges  $0.7 \leq n \leq 1.3$ ,  $-0.3 \leq n_T \leq 0$  and  $0.2 \leq \Omega_0 \leq 1$ , this fit is within 1% almost everywhere, and always within 2.5%. The relation between  $n_T$  and  $r$  is still given by Eq. (13).

- [1] C. L. Bennett et al., *Astrophys. J. Lett.* **464**, L1 (1996).
- [2] M. White and E. F. Bunn, *Astrophys. J.* **450**, 477 (1995) [Erratum **460**, 1071 (1996)]; E. F. Bunn and M. White, Berkeley preprint (1996), astro-ph/9607060.
- [3] A. A. Starobinsky and J. Yokoyama, Kyoto preprint (1995), gr-qc/9502002; J. García-Bellido and D. Wands, *Phys. Rev. D* **52**, 6739 (1995); M. Sasaki and E. D. Stewart, *Prog. Theor. Phys.* **95**, 71 (1996); J. García-Bellido and D. Wands, *Phys. Rev. D* **53**, 5437 (1996); T. T. Nakamura and E. D. Stewart, Tokyo preprint (1996), astro-ph/9604103.
- [4] J. R. Gott, *Nature*, **295**, 304 (1982); M. Sasaki, T. Tanaka, K. Yamamoto and J. Yokoyama, *Phys. Lett. B* **317**, 510 (1993); M. Bucher, A. S. Goldhaber and N. Turok, *Phys. Rev. D* **52**, 3314 (1995); A. D. Linde and A. Mezhlumian, *Phys. Rev. D* **52**, 6789 (1995).
- [5] A. D. Linde, *Phys. Lett.* **129B**, 177 (1983); A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Chur, Switzerland, 1990)
- [6] A. D. Linde, *Phys. Lett. B* **259**, 38 (1991); A. D. Linde, *Phys. Rev. D* **49**, 748 (1994); E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, *Phys. Rev. D* **49**, 6410 (1994).
- [7] B. Whitt, *Phys. Lett.* **145B**, 176 (1984); K. Maeda, *Phys. Rev. D* **39**, 3159 (1989); D. Wands, *Class. Quant. Grav.* **11**, 269 (1994).
- [8] A. R. Liddle and D. H. Lyth, *Phys. Rep.* **231**, 1 (1993).
- [9] E. D. Stewart and D. H. Lyth, *Phys. Lett. B* **302**, 171 (1993).
- [10] A. R. Liddle, P. Parsons and J. D. Barrow, *Phys. Rev. D* **50**, 7222 (1994).
- [11] A. R. Liddle and D. H. Lyth, *Phys. Lett. B* **291**, 391 (1992).
- [12] M. S. Turner, M. White and J.E. Lidsey, *Phys. Rev. D* **48**, 4613 (1993); M. S. Turner and M. White, *Phys. Rev. D* **53**, 6822 (1996).
- [13] E. J. Copeland, E. W. Kolb, A. R. Liddle and J. E. Lidsey, *Phys. Rev. D* **49**, 1840 (1994).
- [14] J. R. Bond, R. Crittenden, R. L. Davis, G. Efstathiou and P. J. Steinhardt, *Phys. Rev. Lett.* **72**, 13 (1994); M. White, *Phys. Rev. D* **53**, 3011 (1996).
- [15] D. H. Lyth, *Phys. Lett.* **147B**, 403 (1984) [Erratum **150B**, 465 (1985)]; S. W. Hawking, *Phys. Lett.* **150B**, 339 (1985); L. M. Krauss and M. White, *Phys. Rev. Lett.* **69**, 869 (1992).
- [16] A. R. Liddle, *Phys. Rev. D* **49**, 739 (1994).
- [17] A. Kosowsky and M. S. Turner, *Phys. Rev. D* **52**, R1739 (1995).
- [18] W. Hu, D. Scott, N. Sugiyama and M. White, *Phys. Rev. D* **52**, 5498 (1995).
- [19] S. M. Carroll, W. H. Press and E. L. Turner, *Ann. Rev. Astron. Astrophys.* **30**, 499 (1992).